GRADE : 12

SUBJECT : Mathematics

TITLE : June Paper 2

EXAMINER : Mr A. Slaughter

TOTAL MARKS : 150

DATE : 5 / 6 / 20 14

SOLUTIONS

TIME : 3 hour(s)

1. 10 13 19 21 22 23 (26) 28 30 30 30 33 34

Q_2 M Q_3

1.1. 1. $\bar{x} = 24,54$ ✔ **1**

1.1. 2. $M = 26$ ✔ **1**

1.1. 3. $\bar{x} - M = 24,54 - 26$

$= -1,46$ ✔

< 0

$\therefore$ data is skewed to the left or negatively skewed. ✔ **2**

1.2. $Q_1 = \frac{19+21}{2}$ $Q_3 = \frac{30+30}{2}$

$= 20$ $= 30$ ✔

$\therefore IQR = 30 - 20$

$= 10$ ✔

$UL = Q_3 + 1,5 . IQR$

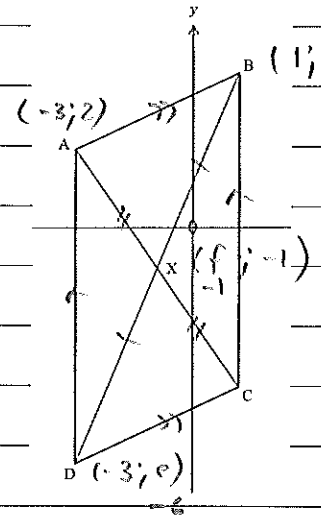
$= 30 + 1,5 . 10$ **3**

$= 45$ ✔

2. Answer Sheet

3. Answer Sheet

4.



4.1. 1. $f = \frac{-3+1}{2} = -1$ ✔ **1**

4.1. 2. $-1 = \frac{e+4}{2}$ ✔

$\times 2: \quad -2 = e + 4$

$-6 = e$ ✔ answer only 2/2 **2**

4.2. 1. $x = -3$ ✔ **1**

4.2. 2. $A(-3;2)$ $B(1;4)$

$CD \parallel AB$ opp sides llgm ✓

$\therefore m_{CD} = m_{AB}$ ✓

$= \frac{4-2}{1-(-3)}$

$= \frac{1}{2}$ ✓

$\therefore y = \frac{1}{2}x + c$

sub $D(-3;-6)$

$-6 = \frac{1}{2}(-3) + c$ ✓

$-\frac{9}{2} = c$

$\therefore y = \frac{1}{2}x - \frac{9}{2}$ ✓ 4

4.3. $A(-3;2)$ $B(1;4)$ $P(k;10)$

$m_{AB} = m_{BK}$

$\frac{1}{2} = \frac{10-4}{k-1}$

method ✓ $\frac{1}{2} = \frac{6}{k-1}$ ✓ m_{BK}

$\therefore k-1 = 12$

$k = 13$ ✓ 3

4.4. $A(-3;2)$ $X(-1;-1)$ $B(1;4)$

$m_{AX} = \frac{-1-2}{1-(-3)} = -\frac{3}{2}$ ✓

$m_{XB} = \frac{4-(-1)}{1-(-1)} = \frac{5}{2}$ ✓

Now, $m_{AX} \cdot m_{XB}$

$= (-\frac{3}{2}) \cdot (\frac{5}{2})$ ✓ product

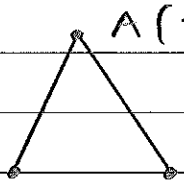
$= -\frac{15}{4}$

$\neq -1$ ✓

$\therefore AX \not\perp XB$

$\therefore$ No ✓ 5

4.5.



$AB = \sqrt{(4-2)^2 + (1-(-3))^2}$

$= \sqrt{20}$ ✓

$AQ = \sqrt{(-1-2)^2 + (y-(-3))^2}$

$= \sqrt{(y+3)^2 + 9}$ ✓

$AQ = AB$

$\sqrt{(y+3)^2+9} = \sqrt{20}$

$()^2$ bs :

$(y+3)^2 + 9 = 20$ ✓

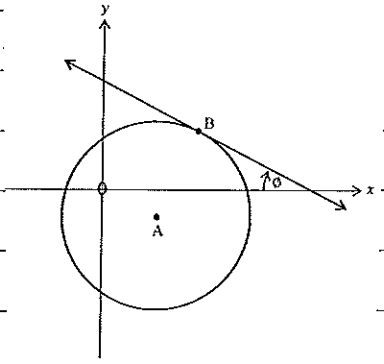
$(y+3)^2 = 11$

$y+3 = \pm\sqrt{11}$ ✓ $^{AB}_{1}$

$\therefore y = -3 \pm \sqrt{11}$

$= 0{,}32$ or $-6{,}32$ ✓ ✓ 6

5.



✓

5.1. $x^2 - 4x + (-2)^2 + y^2 + 2y + (1)^2 = 5 + 4 + 1$

✓$(x-2)^2 + (y+1)^2 = 10$ ✓

$\therefore$ C(2; −1) D 3

5.2. $A_\odot = \pi r^2$

$= \pi . 10$ ✓

$= 31{,}42$ ✓ u^2 D 2

5.3. $m = \tan\theta$

$= \tan(180° - 18{,}43...°)$

$= \tan 161{,}565...°$ ✓

$= -0{,}33...$ ✓

$= -\frac{1}{3}$ ✓ D 3

5.4. $m_{tan} = -\frac{1}{3}$

$\therefore m_{diam} = 3$ tan ⊥ rad

$\therefore$ eqn of diam

$y = 3x + c$ ✓

sub A(2; −1)

$-1 = 3(2) + c$ ✓

$-7 = c$

$\therefore y = 3x - 7$ ✓ ... 1

$x^2 - 4x + y^2 + 2y - 5 = 0$ ... 2

$x^2 - 4x + (3x-7)^2 + 2(3x-7) - 5 = 0$ ✓

$x^2 - 4x + 9x^2 - 42x + 49 + 6x - 14 - 5 = 0$

$10x^2 - 40x + 30 = 0$

$\div 10$: $x^2 - 4x + 3 = 0$ ✓

$(x-1)(x-3) = 0$ ✓

$\therefore x = *$ or 3 ✓2

$\therefore y = 3(3) - 7$

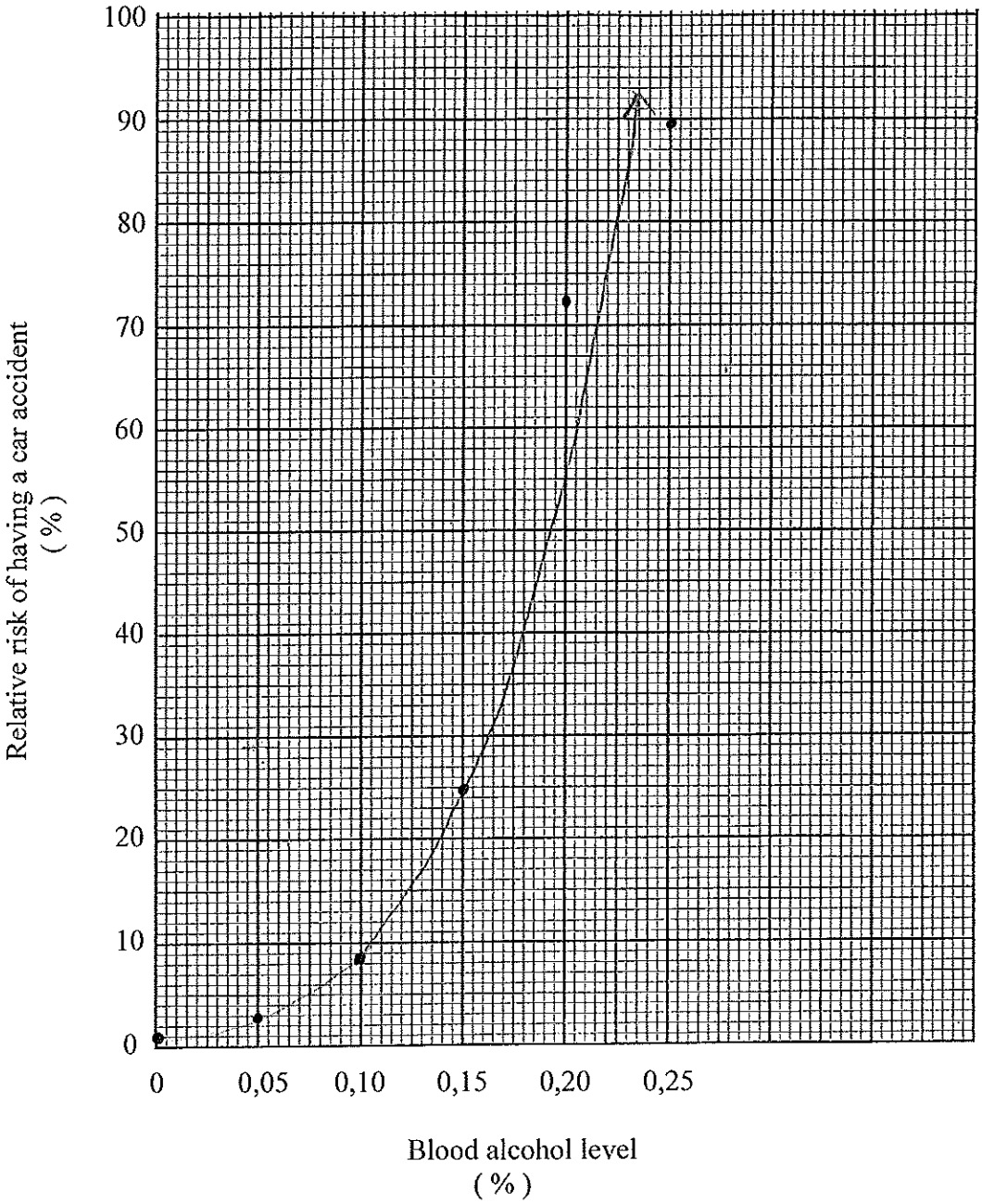
$= 2$

So, B(3; 2) ✓ ✓ D 8

ANSWER SHEET FOR QUESTION 2

2.1.

Scatter plot of Relative risk of having a car accident versus Blood alcohol level



plotting - 1 each error or omission ✓✓

curve ✓ 1

2

2.2. See diagram above.

2.3. As the blood alcohol level increases, the risk of having an accident increases in an exponential manner. ✓ ✓ 2

1s x ↑, y ↑

manner

accept quadratic parabolic

ANSWER SHEET FOR QUESTION 3

3.1.

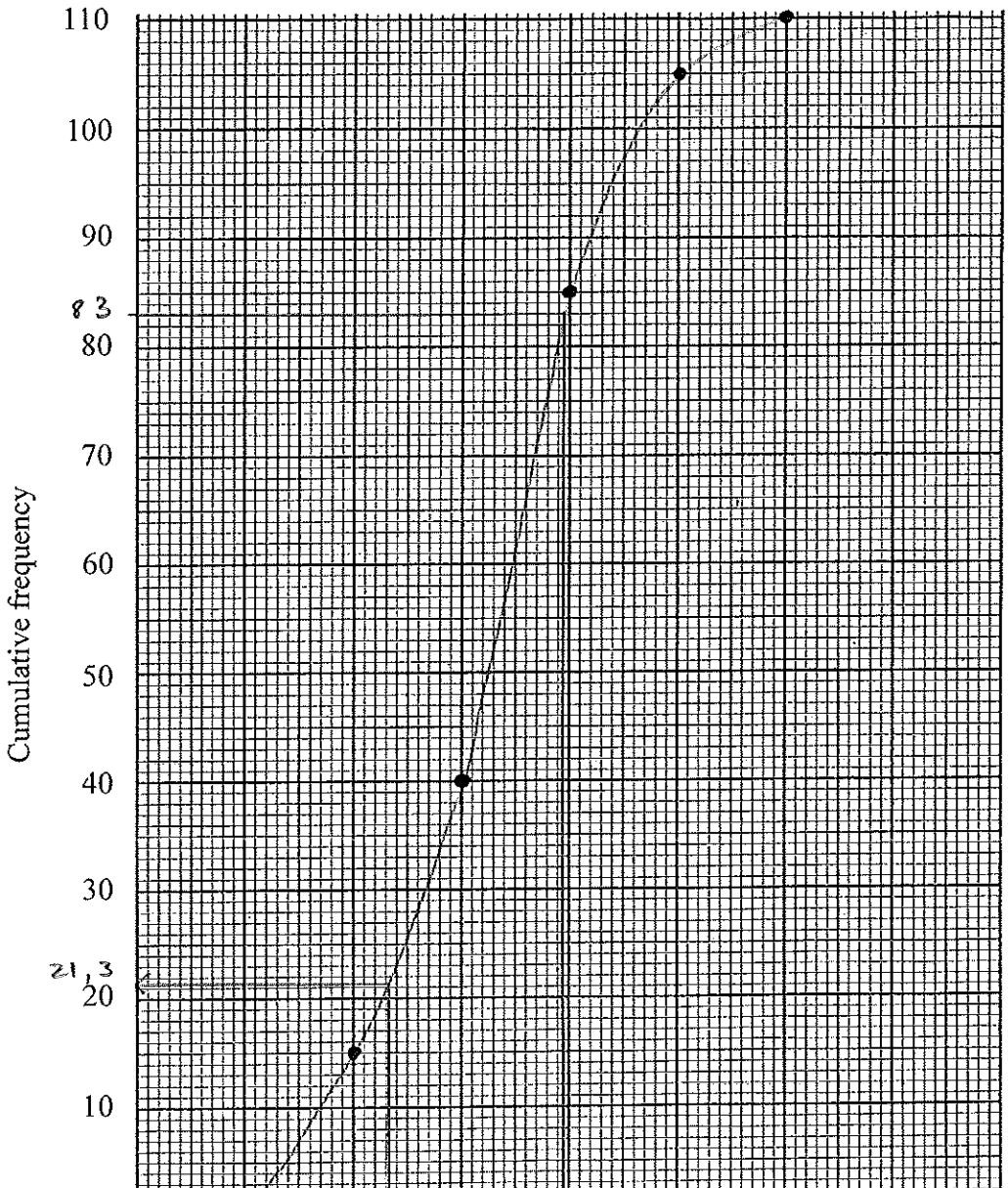
(3;0)

Time taken by people to leave the auditorium (minutes)	Number of people	Cumulative frequency
$3 < x \leq 6$	15	15
$6 < x \leq 9$	25	40
$9 < x \leq 12$	45	85
$12 < x \leq 15$	20	105
$15 < x \leq 18$	5	110

✓ 1

3.2.

Ogive curve of Time taken by people to leave an auditorium



✓ plotting

✓ (3;0) grounding point

✓ smooth curve

3

10 divisions = 3 mins

÷ 10 ÷ 3

1 division = $\frac{3}{10}$ mins = 0,3 mins

or $\frac{10}{3}$ divisions = 1 min

3,33 divisions =

3.3. ≤ 7 min $\longrightarrow$ 21,3 ✓ people

$\therefore > 7$ min $\longrightarrow$ 110 − 21,3

$= 88,7$

$\approx$ 89 ✓ people ▷ 2

no clear indication −1

3.4.1. $T_1; \ldots; T_{110} \therefore M = T_{\frac{1}{2}(1+110)} = T_{55,5}$

$\therefore T_1; \ldots; T_{55} \quad T_{56}; \ldots; T_{110}$

$Q_3 = T_{\frac{1}{2}(56+110)}$

$= T_{83}$ 1

$\therefore$ position 83 ✓ ▷

3.4.2. $\approx$ 11,85 minutes ✓ ▷ 1

3.5. stat mode frequency column or

use : lower, middle or upper end

of interval

$\sigma = 3,12$ ✓✓ or 0 ▷ 2

6.1. $\sin(x-y)$

$= \cos(90^\circ - (x-y))$ ✓

$= \cos(90^\circ - x + y)$

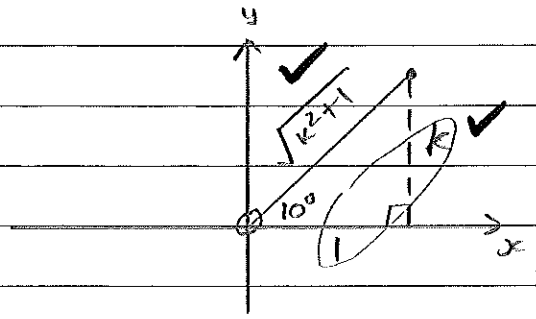
$= \cos((90^\circ - x) - (-y))$ ✓

✓ $= \cos(90^\circ - x)\cos(-y) + \sin(90^\circ - x)\sin(-y)$

$= (+\sin x)(+\cos y) + (+\cos x)(-\sin y)$

$= \sin x \cos y - \cos x \sin y$ → **3**

6.2. $\tan 10^\circ = k$ $\quad \frac{k}{1} \quad \frac{y}{x}$



$\cos 2A = 2\cos^2 A - 1$

$A = 5^\circ$

$\cos 10^\circ = 2\cos^2 5^\circ - 1$ ✓

$\frac{x}{r} = 2\cos^2 5^\circ - 1$

$\frac{1}{\sqrt{k^2+1}} = 2\cos^2 5^\circ - 1$ ✓

$1 + \frac{1}{\sqrt{k^2+1}} = 2\cos^2 5^\circ$

$\frac{1 + \frac{1}{\sqrt{k^2+1}}}{2} = \cos^2 5^\circ$

$\sqrt{\frac{1 + \frac{1}{\sqrt{k^2+1}}}{2}} = \cos 5^\circ$ ✓

→ **5**

(reject $-\sqrt{\ }$)

6.3. $\cos(-234^\circ)$

$= +\cos 234^\circ$

$= \cos(270^\circ - 36^\circ)$

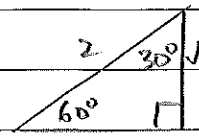
$= -\sin 36^\circ$ ✓

$1 - 2\sin^2 15^\circ$

$= \cos 30^\circ$ ✓

$= \frac{a}{h}$

$= \frac{\sqrt{3}}{2}$ ✓



no mark if diagram not shown

$\sin 18^\circ \cos 18^\circ \times \frac{2}{2}$

$= \frac{2\sin 18^\circ \cos 18^\circ}{2}$

$= \frac{\sin 36^\circ}{2}$ ✓

$\therefore \frac{-\sin 36^\circ}{\frac{\sqrt{3}}{2} \cdot \frac{\sin 36^\circ}{2}}$

$= -\frac{1}{\frac{\sqrt{3}}{4}}$

$= -1 \times \frac{4}{\sqrt{3}}$

$= -\frac{4}{\sqrt{3}}$ ✓ → **5**

7.1. $\frac{1+\tan\theta}{1-\tan\theta} = \frac{1+\sin 2\theta}{\cos 2\theta}$

LHS

$= \frac{1+\frac{\sin\theta}{\cos\theta}}{1-\frac{\sin\theta}{\cos\theta}}$ ✓

$= \frac{\frac{\cos\theta+\sin\theta}{\cos\theta}}{\frac{\cos\theta-\sin\theta}{\cos\theta}}$ ✓

$= \frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta} \times \frac{\cos\theta}{\cos\theta-\sin\theta}$

$= \frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}$ ✓

RHS

$= \frac{1+2\sin\theta\cos\theta}{\cos^2\theta-\sin^2\theta}$ ✓

$= \frac{\sin^2\theta+\cos^2\theta+2\sin\theta\cos\theta}{\cos^2\theta-\sin^2\theta}$ ✓

$= \frac{\cos^2\theta+2\sin\theta\cos\theta+\sin^2\theta}{\cos^2\theta-\sin^2\theta}$

$= \frac{(\cos\theta+\sin\theta)(\cos\theta+\sin\theta)}{(\cos\theta+\sin\theta)(\cos\theta-\sin\theta)}$ ✓

$= \frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}$

$\therefore$ LHS = RHS ▷ **8**

7.2. 1. $\sin(3x-10^\circ)+\cos 2x = 0$

$A = 3x-10^\circ \quad B = 2x$

$\sin A + \cos B = 0$

$\sin A = -\cos B$

$\sin(270^\circ - B) \quad \sin(270^\circ + B)$

• $\sin A = \sin(270^\circ - B)$

$\therefore A = 270^\circ - B + k360^\circ$

$3x-10^\circ = 270^\circ - 2x + k360^\circ$ ✓

$5x = 260^\circ + k360^\circ$ ✓

$x = 56^\circ + k72^\circ$ ✓

or ▷

• $\sin A = \sin(270^\circ + B)$

$A = 270^\circ + B + k360^\circ$

$3x-10^\circ = 270^\circ + 2x + k360^\circ$ ✓

$x = 280^\circ + k360^\circ$ ✓

$(k \in \mathbb{Z})$ ✓ ▷ **5**

7.2. 2. $\sin 2x + 3\cos 2x = 0$

$A = 2x$

$\sin A + 3\cos A = 0$

$\div \cos A$: $\tan A + 3 = 0$

$\tan A = -3$ ✓

$\text{Ref}^\angle = 71,58\ldots^\circ$

tan - an

II: $A = 108,43\ldots + k180^\circ$

$2x = 108,43\ldots + k180^\circ$

$x = 54,22^\circ + k90^\circ$ ✓ ▷ **3**

$(k \in \mathbb{Z})$

ANSWER SHEET FOR QUESTION 8

8.1. $-\sin$ (90°) $x = 90°$ $\therefore$ $x - 90° = 0$

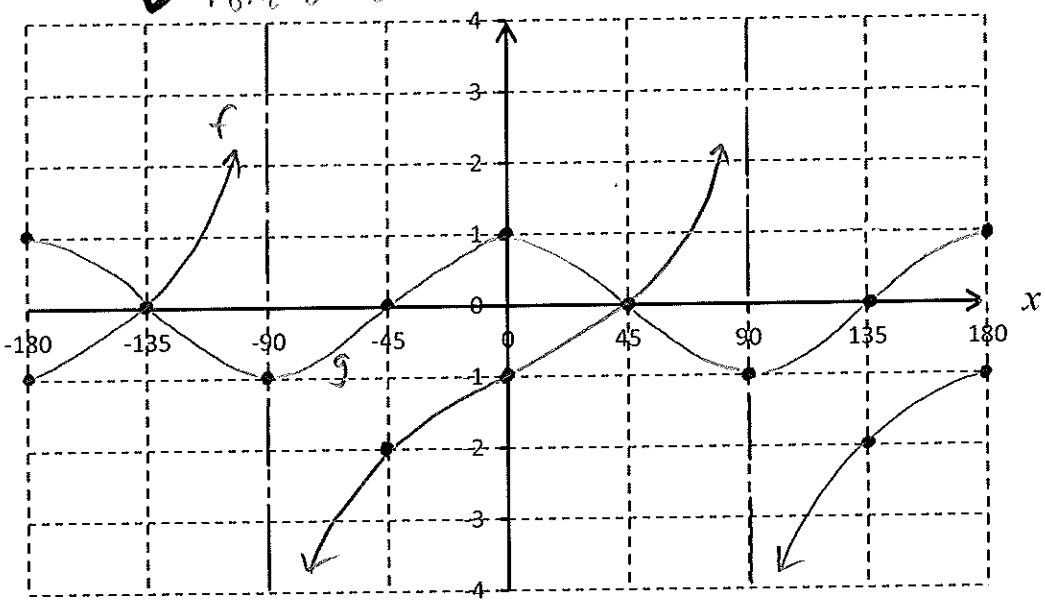
$x + m = 0$

$m = -90°$ ✓ 1

8.2.1. $f(x) = \tan x - 1$ and $g(x) = \cos 2x$

tan ✓ left branch ✓ middle branch ✓ right branch

cos ✓ x int's ✓ y int ✓ amp



6

8.2.2. $\cos 2x \tan x - \cos 2x \geq 0$

✓ $\cos 2x (\tan x - 1) \geq 0$

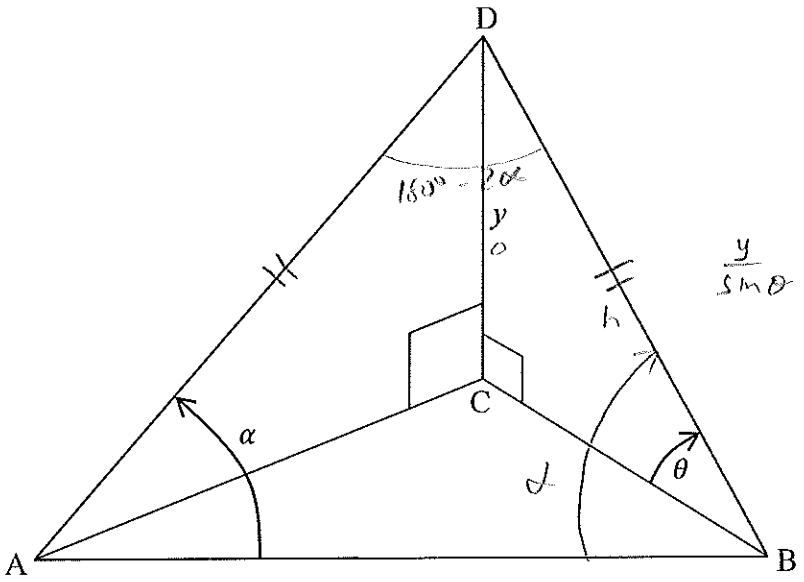
$(y_f)(y_g) \geq 0 \quad + \; 0$

✓ ✓ ✓

$x = 45°$ or $x \in (90°; 135°]$ 4

ANSWER SHEET FOR QUESTION 9.1.

9.1.



9.1.1 In ΔDCB:

$\sin\theta = \frac{y}{DB}$ ✓

$\therefore DB \cdot \sin\theta = y$

$\therefore DB = \frac{y}{\sin\theta}$ ✓ ——▷

2

9.1.2 In ΔADB

$\hat{ABD} = \alpha$ isos Δ, sides =

$\therefore \hat{ADB} = 180° - 2\alpha$ $\angle$'s $\Delta = 180°$

$\therefore \frac{AB}{\sin(180° - 2\alpha)} = \frac{BD}{\sin\alpha}$ ✓

$\therefore AB = \frac{BD \cdot \sin(180° - 2\alpha)}{\sin\alpha}$ ✓

$= \frac{BD \cdot \sin 2\alpha}{\sin\alpha}$ ✓

$= \frac{BD \cdot 2\sin\alpha\cos\alpha}{\sin\alpha}$ ✓

$= BD \cdot 2\cos\alpha$

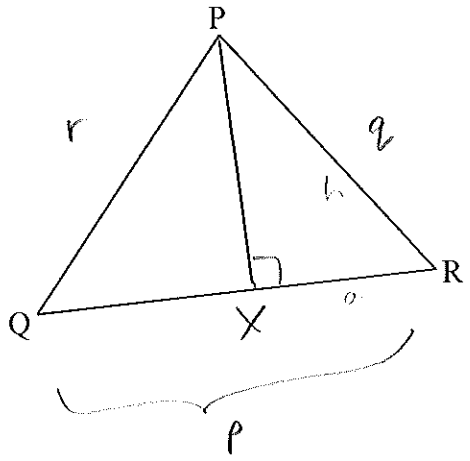
$\therefore AB = \left(\frac{y}{\sin\theta}\right) 2\cos\alpha$ ✓

$= \frac{2y\cos\alpha}{\sin\theta}$ ——▷

5

ANSWER SHEET FOR QUESTION 9.2.

9.2.



In ΔPXR : $\cos\hat{R} = \frac{XR}{q}$

$\therefore XR = q\cos\hat{R}$ ✓

$\therefore QX = p - XR$

$= p - q\cos\hat{R}$ ✓

In ΔPQX : $\cos\hat{Q} = \frac{QX}{r}$ ✓

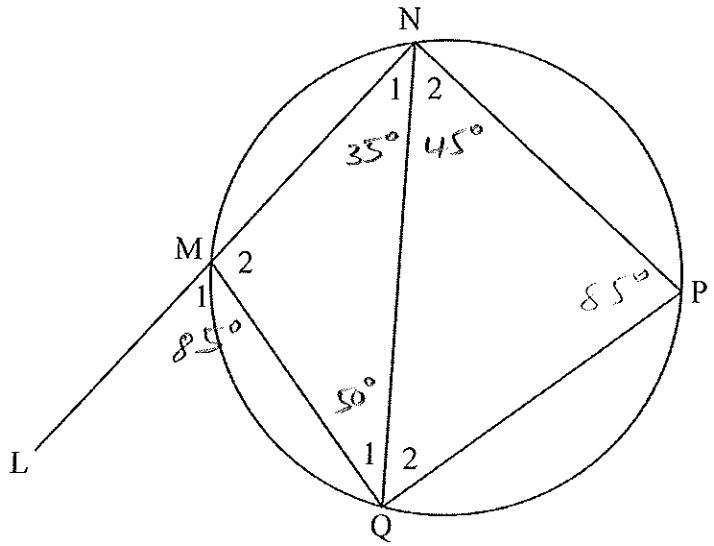
$= \frac{p - q\cos\hat{R}}{r}$

D

3

ANSWER SHEET FOR QUESTION 10

10.1.



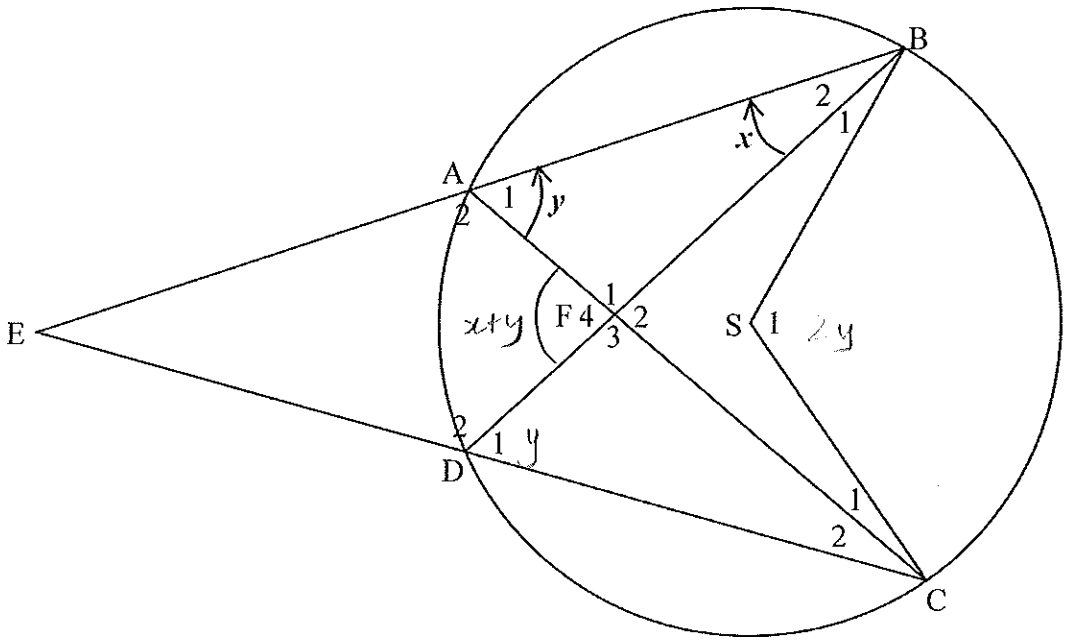
10.1.1. $\hat{M}_1 = 85°$ ✓S ✓R Ext ∠ of △ 2

10.1.2.1. $\hat{P} = 85°$ ✓S Ext ∠ cyclic quad ✓R

$\therefore \hat{Q}_2 = 50°$ ✓SR ∠'s of △ = 180° 3

10.1.2.2. Equal chords are subtended by equal angles ✓ 1

10.2.



10.2.1. $\hat{DFA} = x+y$ ✓SR ✓P Ext ∠ Δ 1

10.2.2. Prove $\hat{S}_1 = F_4 + \hat{E}$

$\hat{D}_1 = y$ ✓S ∠'s same ⊙ segment ✓R

$\hat{S}_1 = 2y$ ✓S ∠ at centre = 2 ∠ @ ⊙'ce ✓R 6

$\hat{E} + x = y$ Ext ∠ Δ

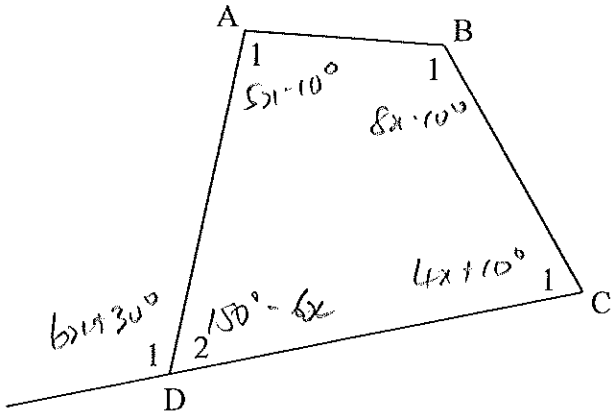
$\therefore \hat{E} = y - x$ ✓SR

$\therefore F_4 + \hat{E} = x+y + y - x$ ✓method

$= 2y$

$= \hat{S}_1$ ⇒

10.3.



10.3.1. $\hat{D}_2 = 180° - (6x + 30°)$ ∠'s str line = 180°

$= 180° - 6x - 30°$

$= 150° - 6x$ ✓ sf

$5x - 10° + 8x - 10° + 4x + 10° + 150° - 6x = 360°$ ✓ s ✓ R ∠'s quad = 360°

$\therefore 11x = 220°$

$x = 20°$ ✓

$\therefore \hat{A}_1 = 5(20°) - 10° = 90°$ ✓

$\therefore \hat{C}_1 = 4(20°) + 10° = 90°$ ✓

$\therefore \hat{A}_1 + \hat{C}_1 = 90° + 90°$

$= 180°$ ✓

$\therefore$ ABCD is a cyclic quad ✓ R conv opp ∠'s cyclic quad = 180° 8

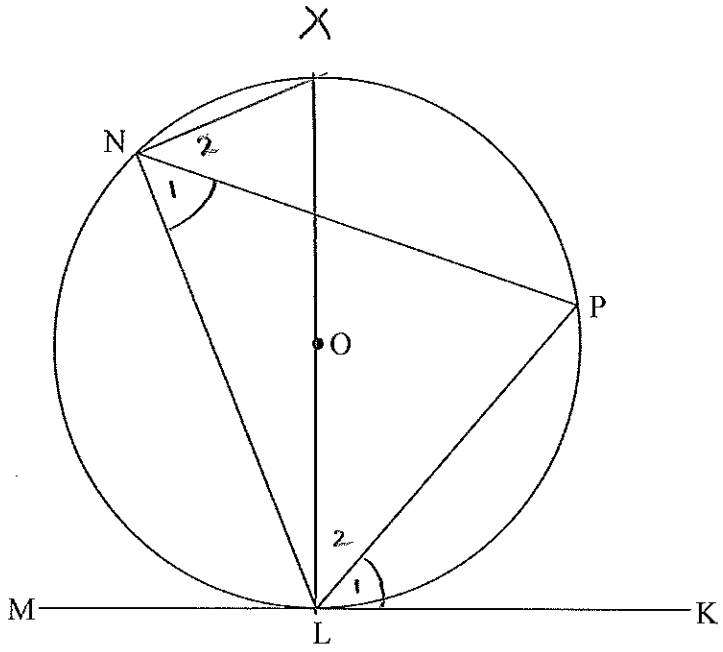
✓ S

10.3.2. BD is diameter of circle through points A, B, C and D. ✓ R 2

$\hat{A}_1 = \hat{C}_1 = 90°$ converse ∠ in a semi ⊙ = 90°.

ANSWER SHEET FOR QUESTION 11

11.1.



Constr : Diam LOX , NX ✓

✓S ✓R

$\hat{L}_1 + \hat{L}_2 = 90°$ ✓S ∠ in semi ⊙ = 90° ✓R

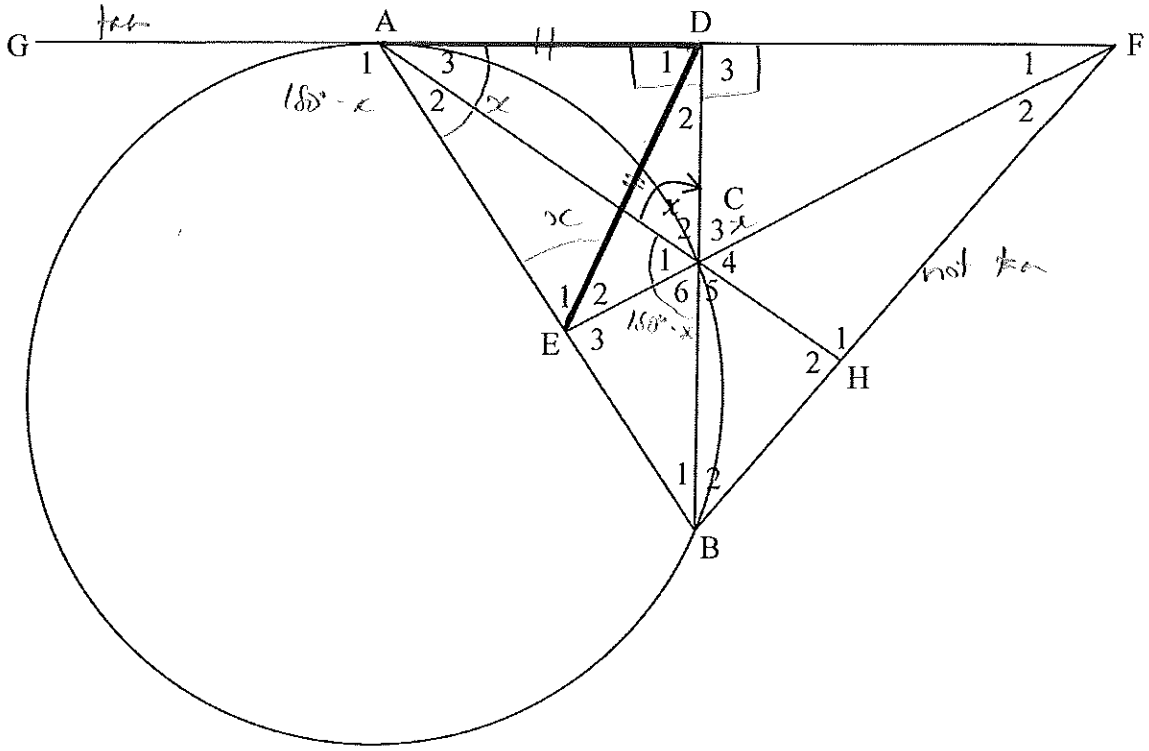
$\hat{N}_1 + \hat{N}_2 = 90°$ ✓S

but $\hat{N}_2 = \hat{L}_2$ ✓S ✓R ∠'s same ⊙ segment

$\therefore \hat{L}_1 = \hat{N}_1$

7

11.2.



11.2.1. $\hat{C}_2 = A_{2+3}$? SR

$\hat{C}_{1+6} = 180° - x$ ✓ ∠'s on str line = 180°

$\therefore \hat{A}_1 = 180° - x$ ✓S ✓R tan chord

$\therefore \hat{A}_{2+3} = x$ ✓SR ∠'s on str line = 180°

$\therefore \hat{C}_2 = \hat{A}_{2+3}$ ▷ $= x$ 4

11.2.2. $\hat{E}_1 = x$ ✓S ISOS Δ, sides =, given ✓R

$\therefore \hat{E}_1 = \hat{C}_2$ ✓conclusion $= x$

$\therefore$ ADCE is a cyclic quad ▷ (converse ∠'s same ○ segment) ✓ 4

11.2.3. $\hat{C}_3 = x$ ✓ ✓Ext ∠ cyclic quad

$\therefore \hat{C}_2 = \hat{C}_3$ ▷ $= x$ 2